Warm Inflation With A General Form Of The Dissipative Coefficient

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Abstract

We propose and investigate a general form of the dissipative coefficient $\Gamma = C_\phi T^m/\phi^{m-1}$ in warm inflation. We focus on discussing the strong dissipative processes $r = \Gamma/3H \gg 1$ in the thermal state of approximate equilibrium. To this toy model, we give the slow-roll conditions, the amplitude and the index of the power spectrum under the general form of dissipative coefficient. Furthermore, the monomial potential and the hybrid-like potential are analyzed specifically. We conclude that the $m = 0, 3$ cases are worthy further investigation especially.

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I. INTRODUCTION

Since the introduction of inflation scenario [1, 2], inflation has been recognized as an essential stage in the early universe. In most inflation models, the scalar field potentials are responsible for accelerating the universe, and the primeval radiation would be red-shifted. The consequence is that the universe becomes cold and we can safely neglect the effect from the temperature. Therefore, the process called "reheating", is needed to make the universe hot again after inflation. From the observational aspect, up to now, we still lack strong evidences to support the point that the universe is cold during inflation. From the theoretical aspect, there also exist some controversy about the temperature of the universe during inflation [3]. A new type of inflation model called "warm inflation", in which the effects from radiation and temperature are considered during inflation, was proposed by Berera and Fang [4, 5]. In warm inflation scenario, the acceleration of the universe is still driven by the potential energy density of the scalar field which is called inflaton, however, because of the interactions between inflaton and other fields, the radiation can not be red-shifted heavily and the universe is hot during inflation. The recent development about warm inflation is nicely reviewed in [6].

Strictly speaking, in warm inflation scenario, the acceleration of the universe should be driven by the free energy density of the thermal system rather than the ordinary potential energy density of the inflaton ϕ. The Hubble parameter H should be expressed as

$$H^2 = \frac{1}{3m_{pl}^2}(\frac{\dot{\phi}^2}{2} + V(\phi, T) + Ts), \tag{1}$$

where m_{pl} is the planck mass, T and s are the temperature and the entropy density of the thermal system respectively. Here, we must point out that the potential $V(\phi, T)$ in Eq. (1) is the effective finite temperature potential, which is not only the function of ϕ but also the function of temperature T. However, under some special backgrounds, such as SUSY, the finite temperature corrections to the potential $V(\phi, T)$ can be suppressed dramatically, so under these backgrounds we can approximatively use the zero temperature potential $V(\phi)$.

After considering the dissipative interaction, the equation of motion of the inflaton be-

comes

$$\ddot{\phi} + (3H + \Gamma)\dot{\phi} + V_\phi = 0, \tag{2}$$

where Γ is the dissipative coefficient, which is related to the microscopic physics of the interactions, and the subscript ϕ means the partial derivative with respect to ϕ. The energy conservation equation of the radiation gives

$$\dot{\rho_r} + 4H\rho_r = \Gamma\dot{\phi}^2, \tag{3}$$

where the right hand side term $\Gamma\dot{\phi}^2$ is the source of the radiation. The hydrodynamic description of the radiation gives the relationship between its energy density and the universal temperature [7]

$$\rho_r = C_r T^4, \tag{4}$$

where $C_r = \pi^2 g^*/30$, g^* is the effective number of the light degrees of freedom in the universe. In the Minimal Supersymmetric Standard Model (MSSM), $g^* = 228.75$ and $C_r = 75.18$, however, if the effective temperature during inflation is lower than the typical temperature of MSSM, g^* could be smaller than order $\mathcal{O}(10)$, and C_r is of order $\mathcal{O}(1)$. In convenience, we define a dimensionless parameter $r = \Gamma/3H$, then $r > 1$ corresponds to a strong dissipative process, $r < 1$ corresponds to a weak dissipative process, and $r \ll 1$ returns to usual ordinary inflation case. For simplicity, we restrict our discussions under the very strong dissipative case $r \gg 1$.

To quantify the dissipative coefficient, we list some results in different models. And the dissipative coefficient mainly depends on the temperature and the amplitude of the inflaton field. With different decaying mechanism, different forms of Γ should be utilized. The forms of dissipative coefficient can be divided into two classes especially in the low temperature, based on the SUSY background or the non-SUSY background. Firstly, we give out the results in the SUSY background. Following Ref. [8], the interactions are given in the super potential

$$W = g\Phi X^2 + hXY^2, \tag{5}$$

where g and h are coupling constants of order $\mathcal{O}(1)$, Φ, X, Y are super fields, and the scalar components of the super fields are ϕ, χ, y respectively. During inflation, the field y

and its fermionic partner $\tilde{y}$ remain massless; while the mediating field χ gets its mass from the interaction with the inflaton ϕ, $m_\chi \approx g\phi$, and its bosonic partner $\tilde{\chi}$ also has a mass of $m_{\tilde{\chi}} \approx g\phi$. When the dissipation works with an intermediate boson($\phi \to 2y$ and $\phi \to 2\tilde{y}$), at the high temperature where $h^{-1}m_\chi > T \gg m_\chi$, $\Gamma \approx 0.691 g^2 T/h^2$, and at the low temperature where $T \ll m_\chi$, $\Gamma \approx 0.04 g^2 h^4 (g\phi/m_\chi)^4 (T^3/m_\chi^2)$. When the dissipation works with an intermediate fermion ($\phi \to y\tilde{y}$) at the high temperature where $h^{-1}m_\chi > T \gg m_\chi$, $\Gamma \approx 0.97 g^2 T/h^2$; at the low temperature where $T \ll m_\chi$, $\Gamma \approx g^{-2}h^4 T^5/\phi^4$ which can be ignored when the dissipation working with an intermediate boson happen at the same time. Furthermore when the dissipation works with the exponential decay propagator, it gives out $\Gamma \approx h^2 g^3 \phi(2 + g\phi/m_\chi + g^3\phi^3/m_\chi^3)/16\pi^2$ in the zero-temperature limit [9, 10, 11]. Secondly, in the non-supersymmetry background, the dissipative coefficient Γ has such a form $\Gamma \approx C_\phi \phi^2/T$ [12, 13], where C_ϕ is $\mathcal{O}(1)$, when there is only one decaying field and one decaying channel.

Considering all the different forms of dissipative coefficients above, we propose such a general form

$$\Gamma = C_\phi \frac{T^m}{\phi^{m-1}}, \tag{6}$$

where C_ϕ is connected to the dissipative microscopic dynamics, and except in the case of $m = 0$ where C_ϕ is an order $\mathcal{O}(10^{-2})$ parameter, we can take C_ϕ to be of order $\mathcal{O}(1)$, if there was only one field and one channel to decay. With the number of the decaying fields and their channels increasing, C_ϕ becomes very large, and a natural upper bound could be $C_\phi < 10^4$ [14]. From the discussions above, we can see that, when $m = -1$, $\Gamma = C_\phi \phi^2/T$, the form corresponds to the dissipative coefficient in the non-SUSY case, when $m = 0$, $\Gamma = C_\phi \phi$, it corresponds to the SUSY case with an exponentially decaying propagator, when $m = 1$, $\Gamma = C_\phi T$, it corresponds to the high temperature SUSY case, when $m = 3$, $\Gamma = C_\phi T^3/\phi^2$, it corresponds to the low temperature SUSY case.

In principle, when the inflaton interacts with other fields, whether the dissipative productions are in an equilibrium process should be determined by the detailed microphysics. However, in our toy model, we just assume that the dissipative coefficient takes the assumed general form that the dissipative process is close to thermal equilibrium, without any calcu-

lations in the microphysics aspect. For convenience, we only focus on the parameter regime $|m| < 4$. And especially, we will pay attention to the cosmological results, focus on the potential takes the monomial form and the hybrid-like form respectively when $m = -1, 0, 1, 3$.

Warm inflation has two main advantages. $\Gamma\dot{\phi}^2$ in Eq. (2) as an additional term compared with ordinary inflation, will make the slow-roll conditions not so restrictive. And for the existence of temperature during inflation, if the thermal fluctuations are much larger than the quantum fluctuations, density perturbations will be reproduced by the thermal fluctuations, rather than the usual quantum fluctuations.

Here we mostly discuss how warm inflation work with a general form of dissipative coefficient $\Gamma = C_\phi T^m/\phi^{m-1}$. We will start with the slow-roll conditions that make sure the inflation phase enough long in Sec. II. And the analytic results for the power spectrum, in particular its amplitude and its index, will be given in Sec. III. For a matter of application, we will focus on two specific potentials, the monomial potential in Sec. IV and the hybrid-like potential in Sec. V. At last, concise summary will be provided in Sec. VI.

II. SLOW ROLL CONDITIONS

In the framework of warm inflation, the scalar potential is still needed to drive the acceleration phase as in the ordinary inflation. Then we also need the slow-roll conditions [15, 16, 17] which technically neglecting the $\dot{\phi}^2$ and Ts in Eq. (1), $\ddot{\phi}$ in Eq. (2), and $\dot{\rho_r}$ in Eq. (3) will ensure sufficient e-folding number, stability of the system. Due to the existence of finite temperature and the new dissipative coefficient, there are three additional slow-roll parameters β, b, c [15]. The slow-roll parameters can be divided into those related to the field ϕ and those related to the temperature T.

The slow-roll parameters related to ϕ are:

$$\epsilon = \frac{m_{pl}^2}{2}(\frac{V_\phi}{V})^2, \eta = m_{pl}^2\frac{V_{\phi\phi}}{V}, \beta = m_{pl}^2\frac{V_\phi \Gamma_\phi}{V\Gamma}, \tag{7}$$

and the warm inflation scenario requires the slow-roll conditions $\epsilon, |\eta|, |\beta| \ll 1 + r$. In the strong dissipative regime $r \gg 1$, the conditions can be simplified to

$$\epsilon, |\eta|, |\beta| \ll r. \tag{8}$$

It shows, just as we mentioned before, that the slow-roll conditions for ϵ and η are much looser than those in ordinary inflation.

The other two additional parameters b and c related to the temperature can be expressed as:

$$b = \frac{TV_{\phi T}}{V_\phi}, c = \frac{T\Gamma_T}{\Gamma} \tag{9}$$

$$0 < b \ll \frac{r}{1+r}, |c| < 4, \tag{10}$$

where the subscript T means partial derivative with respect to T. The parameter b, which shows the extent that the scalar potential is affected by the interactions, can satisfy Eq. (10) if a mechanism for suppressing the thermal corrections is known in the dissipative processes. As we mentioned in the introduction, such dissipative processes, in which the temperature corrections to effective potential are suppressed, can be realized in SUSY background [11, 14, 19]. However, when $m = -1$, there are still some problems for the realization of warm inflation under the SUSY background [12]. In this paper, we will not discuss the specific dissipative mechanism, and we just assume that there is certain mechanism which makes b appropriate for the slow roll condition for simplicity.

With slow-roll conditions, we can ignore the gravitational friction term $3H\dot{\phi}$ and the second order derivative term $\ddot{\phi}$ in Eq. (2), then

$$\dot{\phi} = -\frac{V_\phi}{\Gamma}. \tag{11}$$

Considering Eqs. (4) and (11), the temperature of the universe during inflation is

$$T = (\frac{V_\phi^2 \phi^{m-1}}{4HC_\phi C_r})^{1/(4+m)}. \tag{12}$$

The parameter β gives a new constraint on the evolution of Γ. Combined Eq. (7) with Eq. (12), β can be rewritten as

$$\beta = \frac{2m}{4+m}\eta - \frac{m}{4+m}\epsilon - \frac{4(m-1)}{4+m}\frac{m_{pl}}{\phi}\sqrt{2\epsilon}. \tag{13}$$

If the term m_{pl}/ϕ depends on ϵ or η as we will discuss in Sec. IV and Sec. V, β can be expressed in terms of ϵ and η.

In our case, the parameter c which gives another new constraint on the evolution of Γ, can give a strong constraint on the potential. By virtue of Eq. (12), Eq. (9) can be rewritten as

$$c = m + \frac{(4+m)(1-m)}{m - 1 + (2\phi V_{\phi\phi})/V_\phi - (\phi V_\phi)/(2V)}. \tag{14}$$

In addition, in warm inflation, the potential of the scalar field still dominates the universe during inflation, so $\rho_r/V < 1$ is the basic requirement, furthermore the workable temperature regions are crucial too. Following, we will discuss the value of ρ_r/V and the value of T/ϕ in specific models.

III. POWER SPECTRUM

For a scalar field, if $T > H$, the amplitude of thermal fluctuations will be larger than that of quantum fluctuations [5], then the main contribution to the energy density perturbations comes from the thermal fluctuations. The amplitude of the spectrum in the strong dissipative regime ($r \gg 1$) is [17, 18]

$$P_R^{1/2} \simeq |\frac{H}{\dot{\phi}}|(\frac{\pi r}{4})^{1/4}\sqrt{TH} = |\frac{3H^3}{V_\phi}|(\frac{\pi r}{4})^{\frac{1}{4}}(1+r)\sqrt{\frac{T}{H}}. \tag{15}$$

The observations tell us that the order of the amplitude of power spectrum should be $P_R^{1/2} \simeq 5 \times 10^{-5}$. Compared with ordinary inflation, the amplitude of power spectrum in warm inflation does not only depend on the Hubble parameter H and the potential V but also depend on the dissipative coefficient Γ which is contained in r and the temperature T. In Eq. (15), large T and r are not favored by a small amplitude of the spectrum, and we will discuss this problem in the concrete models in detail.

We can calculate the index of the spectral in the strong dissipative regime by using the definition

$$n_s - 1 = 2\frac{d\ln P_R^{1/2}}{d\ln k} = 2\frac{\dot{\phi}}{H}\frac{d\ln P_R}{d\phi} = \frac{-2V_\phi}{3H^2(1+r)}\frac{d\ln P_R^{1/2}}{d\phi}, \tag{16}$$

where k is the wavenumber, and that combined with Eqs. (6), (12) and (15), we can get

$$\begin{aligned} n_s - 1 \;=\; & \frac{-1}{1+r}[\frac{-6-2m+mC}{4+m}\eta + \frac{(-2-m)C+19+5m}{4+m}\epsilon \\ & + \frac{(-m+1)(-1+2C)}{4+m}\frac{m_{pl}}{\phi}\sqrt{2\epsilon}], \end{aligned} \tag{17}$$

where

$$C=\frac{(8+m)(r+1)+4r(3+m)}{(4+m)(r+1)-2m}. \tag{18}$$

As $r \gg 1$, $C \approx 5$. Further simplifying Eq. (17), we can get

$$n_s-1\approx\frac{-1}{1+r}[\frac{-6+3m}{4+m}\eta+\frac{9}{4+m}\epsilon+\frac{9(1-m)}{4+m}\frac{m_{pl}}{\phi}\sqrt{2\epsilon}]. \tag{19}$$

And if we rewrite the above equation, according to Eq. (13) we can get

$$n_s-1\approx\frac{1}{1+r}(\frac{2}{3}\eta-\frac{9}{4}\epsilon-\frac{9}{4}\beta) \tag{20}$$

which agrees with the previous results in [14, 15, 19, 20]. The WMAP5 data require $n_s \simeq 1$ for a near flat power spectrum, more specifically, $0.949 < n_s < 0.977$ if the running of the spectral is forbidden, and $0.976 < n_s < 1.085$ with a running of the spectral $-0.065 < dn_s/d\ln k < -0.009$ [21]. The running of the spectral can be expressed as

$$\begin{aligned} n_s' &= \frac{dn_s}{dlnk}=\frac{1-n_s}{1+r}r'-\frac{1}{1+r}[\frac{-6+3m}{4+m}\eta'+\frac{9(1-m)}{4+m}(\frac{m_{pl}}{\phi}\sqrt{2\epsilon})' \\ &\quad +\frac{9}{4+m}\epsilon'+\frac{mD}{4+m}\eta+\frac{2(1-m)D}{4+m}\frac{m_{pl}}{\phi}\sqrt{2\epsilon}-\frac{(2+m)D\epsilon}{4+m}], \end{aligned} \tag{21}$$

where

$$D=C'\approx\frac{3+m}{4+m}\frac{5r'}{r},$$

$$\eta'=-\frac{\epsilon}{1+r}(\xi-2\eta), \tag{22}$$

$$\epsilon'=-\frac{2\epsilon}{1+r}(\eta-2\epsilon), \tag{23}$$

$$(\frac{m_{pl}}{\phi}\sqrt{2\epsilon})'=\frac{m_{pl}}{\phi}\frac{\sqrt{2\epsilon}}{1+r}(\frac{m_{pl}}{\phi}\sqrt{2\epsilon}+2\epsilon-\eta), \tag{24}$$

$$r'=\frac{[(4+2m)\epsilon+(4m-4)\sqrt{2\epsilon}m_{pl}/\phi-2m\eta]r}{(4+m)r+4-m}, \tag{25}$$

where $' = d/d\ln k$, $\xi = 2m_{pl}^2 V_{\phi\phi\phi}/V_\phi$. When $dn_s/d\ln k < 0$, the running of the spectrum is negative, and relatively when $dn_s/d\ln k > 0$, the running of the spectrum is positive. From Eq. (17) if both $\epsilon \sim \eta$ and $m_{pl}\sqrt{2\epsilon}/\phi \sim \eta$, $n_s - 1$ will be of order $\mathcal{O}(\eta/r)$, and $dn_s/d\ln k$ will be of order $\mathcal{O}(\eta^2/r^2)$. And if η is of order $\mathcal{O}(1)$, as $r \gg 1$, we will probably get a near flat spectrum. The η problem which can not get a flat power spectrum when η at order $\mathcal{O}(1)$ in

the ordinary inflation is alleviated. When the slow-roll conditions in Eq. (8) are satisfied, the spectrum is probably flat and the running could be small. As we see, the expression of the running is very complicated. In concrete potentials, we will use the exact form of n_s to carry on our calculations.

As in Ref. [15], the power spectrum of tensor perturbations is just the same as in the ordinary inflation $P_T = H^2/(2\pi^2 m_{pl}^2)$, so the tensor-to-scalar amplitude ratio is

$$\frac{P_T}{P_R} = \frac{2}{\pi^{5/2}} \frac{\epsilon}{(1+r)^2 r^{1/2}} \frac{H}{T}. \tag{26}$$

If $\epsilon/r < 1$, as $T/H > 1$ and $r \gg 1$, the tensor-to-scalar amplitude ratio will be much smaller than that in the ordinary inflation where $P_T/P_R = 12.4\epsilon$ [22]. This result could probably meet the observations ($P_T/P_R < 0.43$ without a running of the spectrum, and $P_T/P_R < 0.58$ with a running of the spectrum [21]).

In the next two sections, we will apply the above equations to the monomial potential and the hybrid-like potential respectively. To get a realizable warm inflation, we will discuss about four dominant conditions: the slow-roll conditions $\epsilon, |\eta|, |\beta| \ll r$ and $|c| < 4$; $T/H > 1$ which makes the density spectrum originated from the thermal fluctuations; the observable amplitude of the density perturbations $P_R^{1/2} \approx 5 \times 10^{-5}$ which should be satisfied; the workable temperature regime which should be suitable as well.

IV. MONOMIAL POTENTIAL

The form of monomial potential reads

$$V(\phi) = V_0 (\frac{\phi}{m_{pl}})^n, \tag{27}$$

where V_0 and n are the two free parameters of the theory. As we know, when $n = 2$, it is the well known chaotic inflation, and when $n = 4$, it represents the self interactions of the inflaton. In this paper, we only consider the cases $n \geq 2$ in this kind of potential. In ordinary inflation the workable regime of this kind of models is $\phi > m_{pl}$ in which the causal problem would arise [23]. We put this potential in warm inflation background to check whether the inflation can work when we require $\phi \leq m_{pl}$ and $V(\phi) < m_{pl}^4$ simultaneously.

From Eq. (2), we can get that the potential deceases from its initial value V_i as

$$\frac{V}{V_i}=(1-\frac{4N_e\rho_{ri}}{V_i}\frac{12+mn-2n-4m}{n(4+m)})^{n(4+m)/(12+mn-2n-4m)}, \tag{28}$$

where N_e is the e-folding number with the definition $N_e=\int_t^{t_{end}}Hdt$, the subscript i means the value at the initial time. If $V/V_i \ll 1$, the potential could not dominate the energy density of the universe during inflation. To make the model workable, we must have $V/V_i \sim \mathcal{O}(10^{-1})$.

Since the dissipative coefficient is related to the temperature, it is necessary to calculate the value of T/ϕ for discussing the workable temperature regime. Eqs. (2), (12) and (27) yield

$$\frac{T}{\phi}=[(\frac{\sqrt{3}n^2}{4C_\phi C_r})^2(\frac{V_0}{m_{pl}^4})^3(\frac{\phi_i}{m_{pl}})^{(3n-8-2m)}(\frac{V}{V_i})^{(3n-14)/n}]^{1/2(4+m)}. \tag{29}$$

The evolution of T/ϕ depends on the value of n and m sensitively. Assuming $\phi_i \approx m_{pl}$ and $V_i \ll m_{pl}^4$, we can get $T/\phi < 1$, and the warm inflation works in the low temperature regime. And as Eq. (29) shows, if $\phi_i \ll m_{pl}$, warm inflation may take place in the high temperature regime too when $n < (8+2m)/3$. For convenience, we only consider the $m=-1,0,1,3$ cases in the following.

A. Slow Roll Conditions in Monomial Potential

Considering the slow-roll parameters in the monomial potential, ϵ and β can be expressed in terms of η

$$\epsilon=\frac{n}{2(n-1)}\eta, \tag{30}$$

$$\beta=\frac{3mn-12m+8}{2(n-1)(4+m)}\eta. \tag{31}$$

Then as $n \geq 2$, $|m|<4$, we can get $\epsilon \leq \eta$, β is of order $\mathcal{O}(\eta)$. And we only need to check whether $|\eta/r|<1$ while considering the slow-roll parameters related to ϕ. In this potential $\eta>0$, $|\eta/r|=\eta/r$, we will use $\eta/r<1$ below. In strong dissipation, from Eqs. (12) and (27), we can get

$$\eta=n(n-1)(\frac{V_i}{V})^{2/n}(\frac{m_{pl}}{\phi_i})^2, \tag{32}$$

$$r=[\frac{C_\phi^4}{9}(\frac{n^2}{4C_r})^m(\frac{m_{pl}}{\phi_i})^{-4+6m}(\frac{V_i}{m_{pl}^4})^{m-2}(\frac{V_i}{V})^{(-mn+2n-4+6m)/n}]^{1/(4+m)}, \tag{33}$$

$$\frac{\eta}{r} = 4\frac{n-1}{n}[\frac{9n^8C_r^m}{16C_\phi^4}(\frac{V_0}{m_{pl}^4})^{2-m}(\frac{\phi_i^n}{m_{pl}^n}\frac{V}{V_i})^{(2n-mn-12+4m)/n}]^{1/(4+m)}. \quad (34)$$

By virtue of Eqs. (28) and (34), $\eta/r < 1$ is equivalent to

$$4^{(m+2)/4}\sqrt{3n^2}C_r^{m/4}(N_e\frac{|12+mn-2n-4m|}{n(4+m)}+1-\frac{1}{n})^{(m+4)/4}(\frac{m_{pl}}{\phi_i})^{3-m}(\frac{V_i}{m_{pl}^4})^{(2-m)/4} < C_\phi, \quad (35)$$

so C_ϕ is bounded below.

From Eqs. (11) and (34), we can get $\rho_r/V = [n/(4n-4)](\eta/r)$, which means that if the slow-roll conditions are satisfied, $\rho_r/V < 1$ will be satisfied automatically.

Then we check the slow-roll parameter c. In monomial potential

$$|c| = |m + \frac{(1-m)(4+m)}{m-3+3n/2}| < 4, \quad (36)$$

which gives a strong constraint on the model parameter n directly. Table I denotes the suitable n which is required by appropriate c when $m = -1, 0, 1, 3$. When $m = -1, 0$ the chaotic case ($n = 2$) is excluded by the constraint on c. And the $n = 4$ case can satisfy the requirements of c when $m = -1, 0, 1, 3$.

TABLE I: The suitable range of n for c, the value of $n_s - 1$, the proper range of n for the red spectrum and the blue spectrum in the monomial potential.

m	$\lvert c\rvert < 4$	$n_s - 1$	red spectrum	blue spectrum
-1	$n > 52/15$	$\frac{-\eta}{1+r}\frac{12-3n}{2(n-1)}$	$2 \leq n < 4$	$n > 4$
0	$n > 8/3$	$\frac{-\eta}{1+r}\frac{21-3n}{8(n-1)}$	$2 \leq n < 7$	$n > 7$
1	$n \geq 2$	$\frac{-\eta}{1+r}\frac{6+3n}{10(n-1)}$	$2 \leq n$	none
3	$n \geq 2$	$\frac{-\eta}{1+r}\frac{-24+9n}{14(n-1)}$	$n > \frac{8}{3}$	$2 \leq n < \frac{8}{3}$

B. Power Spectrum in Monomial Potential

To study the energy density perturbations which come from the thermal fluctuations, we list the analytic form of T/H:

$$\frac{T}{H} = [3^{5+m}(\frac{n^2}{4C_\phi C_r})^2(\frac{\phi_i^n}{m_{pl}^n}\frac{V}{V_i})^{(6-2m-mn-n)/n}(\frac{m_{pl}^4}{V_0})^{1+m}]^{1/2(4+m)}, \quad (37)$$

and the amplitude of the power spectrum as well

$$P_R^{1/2} = (\frac{\pi}{4n^4})^{1/4}[3^{-m+13}C_\phi^{18}(\frac{n^2}{4C_r})^{5m+2}(\frac{V_0}{m_{pl}^4})^{6m-3}(\frac{\phi_i^n}{m_{pl}^n}\frac{V}{V_i})^{(42-28m+6mn-3n)/n}]^{1/4(4+m)}. \quad (38)$$

TABLE II: The value of $dn_s/d\ln k$, the suitable range of n for the negative running and the positive running of the spectra in monomial potential.

m	$dn_s/d\ln k$	negative running	positive running
-1	$\frac{\eta^2}{r^2}\frac{(3n-16)(4-n)}{(n-1)^2}$	$2 \le n < 4$, $n > \frac{16}{3}$	$4 < n < \frac{16}{3}$
0	$\frac{\eta^2}{r^2}\frac{(n-6)(21-3n)}{8(n-1)^2}$	$2 \le n < 6$, $n > 7$	$6 < n < 7$
1	$\frac{\eta^2}{r^2}\frac{(n-8)(6+3n)}{25(n-1)^2}$	$2 \le n < 8$	$n > 8$
3	$\frac{\eta^2}{r^2}\frac{-n(5n-6)}{49(n-1)^2}$	$n > \frac{8}{3}$	$2 \le n < \frac{8}{3}$

Combining Eq. (17) with Eq. (30), the potential gives $nm_{pl}\sqrt{2\epsilon}/\phi = \eta$, and we can yield the index of the spectrum

$$n_s - 1 = \frac{-\eta_H}{1+r}\frac{6mn + 21 - 15m - 3n}{2(m+4)(n-1)}, \quad (39)$$

which leads to the running of the spectrum as

$$\frac{dn_s}{d\ln k} = \frac{\eta^2}{r^2}\frac{(6mn + 21 - 15m - 3n)(2n - mn - 12 + 4m)}{(4+m)^2(n-1)^2}. \quad (40)$$

$n - 1 = 0$ is not a real singularity for the two equations above because in Eq. (34) the η/r contain the $n-1$ term too that can eliminate the singularity. We can see that these formulas depend on m sensitively. Considering that η/r is positive, the scopes of n for both the red spectrum and the blue spectrum are given in Table I, and the proper regions of n for both the negative running and the positive running of the spectrum are all listed in Table II. When $n = 2$, the spectrum is red with a negative running in the $m = -1, 0, 1$ cases, and blue with a positive running in the $m = 3$ case. When $n = 4$, the spectrum is exactly flat in the $m = -1$ case, blue in the $m = 0, 1$ cases, and red in the $m = 3$ case, with no running in the $m = -1$ case and a negative running in the $m = 0, 1, 3$ cases.

We discuss the spectrum in the $m = -1, 0, 1, 3$ in detail below. When $m = -1$,

$$\frac{T}{H} = (\frac{9n^2}{4C_\phi C_r})^{1/3}(\frac{\phi_i}{m_{pl}}\frac{V}{V_i})^{4/3}. \quad (41)$$

Assume that C_r and C_ϕ at order $\mathcal{O}(10)$, as $\phi_i/m_{pl} \leq 1$, and with ϕ rolling down, the potential energy of ϕ decreases, $V/V_i < 1$, so the terms on the right hand side of Eq. (41) are not bigger than 1, then $T/H > 1$ is impossible. Therefore warm inflation can not produce an observable power spectrum caused by the thermal fluctuations when $m = -1$ and $T/H < 1$ in the monomial potential.

When $m = 0$, $T/H > 1$ yields

$$\frac{V_0}{m_{pl}^4} < 3^5 (\frac{n^2}{4C_\phi C_r})^2 (\frac{\phi_i^n}{m_{pl}^n} \frac{V}{V_i})^{(6-n)/n}. \tag{42}$$

Combining the above expression with Eq. (38), we can get

$$\frac{\phi_i}{m_{pl}} < \left[0.49 n^{-14} C_\phi^{15} C_r^5 (\frac{V}{V_i})^{3/2n}\right]^{3/16} P_R^{1/3}. \tag{43}$$

If we assume the terms in the square brackets are at order $\mathcal{O}(1)$, we will get $\phi_i/m_{pl} < 10^{-3}$ which is much different from the ordinary inflation case where $\phi_i/m_{pl} > 1$, so in this model we can avoid the causal problem. Further considering the parameter V_0, taking $n = 2$ for example, we can get $V_0^{\frac{1}{4}} < 10^{15} GeV$ from Eq. (42). In addition, $\eta/r < 1$ requires the amplitude of the spectrum $P_R^{1/2} > 6(n-1)^{9/8}(n/C_r)^{1/8}(\phi_i/m_{pl})^{(3n-6)/8}(V_0/m_{pl}^4)^{3/8}$. If the parameter value is as small as we discussed before, it is easy to satisfy this relation, and the $m = 0$ case can work in the low temperature regime [8]. Combined with Eq. (29), Eq. (42) gives

$$\frac{T}{\phi} < \frac{9n^2}{4C_\phi C_r} (\frac{V}{V_i})^{1/2n} (\frac{\phi_i}{m_{pl}})^{5/4}. \tag{44}$$

If $\phi_i/m_{pl} \leq 1$, T/ϕ could be smaller than 1, the model can also work in the low temperature regime, so that the warm inflation can be realized. And we notice that the above discussions do not give an obvious constraint to the parameter C_ϕ, whether C_ϕ is larger or smaller than 10^4, this scenario can both work.

When $m = 1$, $T/H > 1$ gives

$$\frac{V_0}{m_{pl}^4} < \frac{27n^2}{4C_\phi C_r} (\frac{m_{pl}^n}{\phi_i^n} \frac{V_i}{V})^{(n-2)/n}, \tag{45}$$

and $\eta/r > 1$ gives

$$\frac{V_0}{m_{pl}^4} < 576^{-1} n^{-3} (n-1)^{-5} C_\phi^4 C_r^{-1} (\frac{m_{pl}^n}{\phi_i^n} \frac{V_i}{V})^{(n-8)/n}. \tag{46}$$

If the two conditions are satisfied, they give the amplitude of the spectrum two upper bounds. Because the observation constraints are more strict $P_R^{1/2} \approx 5 \times 10^{-5}$, in fact the above constraints are too loose to be effective.

The analyzed amplitude of power spectrum in this model is:

$$P_R^{1/2} = [1.1n^{-3/10} C_r^{-7/20} C_\phi^{9/10} (\frac{\phi_i^n}{m_{pl}^n} \frac{V}{V_i})^{(14+3n)/20n}] (\frac{V_0}{m_{pl}^4})^{3/20}. \tag{47}$$

We assume $\phi_i \approx m_{pl}$ and the terms in the square brackets are of order $\mathcal{O}(1)$, the observable amplitude of the power spectrum requires $V_0^{\frac{1}{4}} \approx 8 \times 10^{10} GeV$. And as mentioned in Eq. (29), under that assumption the model favors a low temperature case when $m = 1$. But the dissipative processes need a high temperature regime when $m = 1$. Therefore, if we want to realize warm inflation, we need a smaller ϕ_i, not only to get a suitable power spectrum, but also to get a relative high temperature in the $m = 1$ case, but that is only useful when $n < 10/3$ as Eq. (29) shown. And whether C_ϕ is larger or smaller than 10^4 is not a problem for this case too.

When $m = 3$, a very detailed discussion has been done in Ref. [14]. In our calculation, $T/H > 1$ yields

$$\frac{V_0}{m_{pl}^4} < 4.5 (\frac{n^2}{C_\phi C_r})^{1/2} (\frac{V_i}{V})^{1/n} \frac{m_{pl}}{\phi_i}, \tag{48}$$

which is easy to satisfy if we choose a low value of V_0. And $\eta/r < 1$ requires

$$\frac{V_0}{m_{pl}^4} > 9216 n(n-1)^7 C_r^3 C_\phi^{-4} \frac{V_i}{V} (\frac{m_{pl}}{\phi_i})^n. \tag{49}$$

Then we use the above inequality to replace V_0/m_{pl}^4 term in $P_R^{1/2}$, and then we can get

$$P_R^{1/2} > 1.64 n^{7/4} (n-1)^{15/4} C_\phi^{-3/2} C_r (\frac{m_{pl}}{\phi_i})^{3/2} (\frac{V}{V_i})^{-3/2n}. \tag{50}$$

In the above expression, assume $\phi_i/m_{pl} \approx 1$ and C_r is at order $\mathcal{O}(10)$, C_ϕ must be at least at order $\mathcal{O}(10^4)$ to get low enough value of $P_R^{1/2}$, just as Ref. [14] mentioned. If ϕ_i is smaller than m_{pl}, C_ϕ will be larger than $\mathcal{O}(10^4)$ which is too large to be natural. Therefore, we exclude the $m = 3$ case.

The above discussions are based on two assumptions, one is that ϕ is smaller than m_{pl}, which can avoid the causal problem in ordinary inflation, the other is that the remained

parameters also satisfy the requirements from the particle physics including $V_0 < m_{pl}^4$ and a natural choice of the number of the inflaton fields $C_\phi < 10^4$. We find some regimes for monomial potential the warm inflation can work. The $m = 0$ case is workable. When $m = 1$, the regime $2 \leq n < 10/3$ can work as well. And in the $m = -1$ case, the warm inflation can not be realized because $T/H < 1$. In the $m = 3$ case, the constraints from C_ϕ excludes the warm inflation.

V. HYBRID-LIKE POTENTIAL

We now turn to the hybrid-like potential in warm inflation which is particularly motivated from particle physics. During the inflation phase the potential of the hybrid-like inflation is effectively described by a single field,

$$V(\phi) = V_0(1 + \alpha \ln \frac{\phi}{M}), n = 0, \tag{51}$$

$$V(\phi) = V_0(1 + (\frac{\phi}{M})^n), n > 0, \tag{52}$$

α is the coupling constant, in the Yukawa coupling or the gauge coupling which is of order $\mathcal{O}(1)$, and M is a model parameter which depends on the exact physical mechanism and we just treat as an undetermined parameter. When $n = 0$, the potential is derived from the one-loop correction. And in the $n = 2$ case, the potential presents the standard hybrid-like model. However, there are some constraints on the parameters from the point of view of particle physics. Firstly, since the physics above the Planck scale is unknown, the scale of the potential should be $V(\phi) < m_{pl}^4$. Secondly, when $n \geq 4$, the parameters must satisfy the condition $V_0/(M^n m_{pl}^{4-n}) < 1$, which means that the coupling constant should not be too large to make the field ϕ ill-defined [22]. On the other hand $V_0/(M^n m_{pl}^{4-n})$ should not be too small to make the coupling constant fine-tuning. In this paper, we assume $\phi \ll M$, $\phi \leq m_{pl}$ simultaneously and V_0 is the main contribution to H^2 where $H^2 \approx V_0/3m_{pl}^2$. As inflation continues, the field ϕ decreases during inflation.

We define a dimensionless parameter a for convenience,

$$a^2 = \frac{\alpha V_0}{H^4}, n = 0; \tag{53}$$

$$a^2 = \frac{nV_0}{H^4}(\frac{H}{M})^n, n \neq 0. \tag{54}$$

When $n = 0$, we have $a^2 = 3\alpha m_{pl}^2/H^2$. If $H < m_{pl}$, $a^2 > 3\alpha$ which means a is larger than 1. When $n \geq 4$, $V_0/(M^n m_{pl}^{4-n}) < 1$ yields $a^2 < n(H/m_{pl})^{n-4}$, and as a result when $H < m_{pl}$, a is smaller than 1. When $n = 2$, we just have $a^2 = 6m_{pl}^2/M^2$, which mean a can not be decided.

Because the effective regions of the dissipative parameter depend on the temperature, it is important to investigate how ϕ/T evolves:

$$\frac{\phi}{T} = \frac{\phi_i}{T_i}(\frac{\phi}{\phi_i})^{(7-2n)/(4+m)} = [4C_r C_\phi a^{-4}(\frac{\phi}{H})^{7-2n}]^{1/(4+m)}. \tag{55}$$

As ϕ deceases, $\phi/\phi_i < 1$, the evolution of ϕ/T depends on the value of n, and the value of ϕ/T depends on a and ϕ/H remarkably.

A. Slow roll parameters in hybrid-like potential

As in monomial potential, we first pay attention to the slow-roll parameters. In hybrid-like potential,

$$\epsilon \approx \frac{n^2}{2}\frac{\phi^{2(n-1)}m_{pl}^2}{M^{2n}} = \frac{n\eta}{2(n-1)}(\frac{\phi}{M})^n. \tag{56}$$

As $\phi \ll M$, we have $\epsilon < \eta$, and the slow-roll requirement for ϵ can be easily satisfied as long as $|\eta/r| < 1$.

The parameter β can be written in the form of ϵ and η:

$$\beta = \frac{2m}{4+m}\eta - \frac{m}{4+m}\epsilon - \frac{4(m-1)}{4+m}\frac{\eta}{n-1}. \tag{57}$$

If ϵ and η could satisfy the slow-roll conditions, so could β. Combined with the discussions before, $|\eta/r| < 1$ is the key relation to the slow-roll requirements.

Due to the hybrid-like potential, the parameter $r = [C_\phi^4(a^4/4C_r)^m(\phi/H)^{2nm-6m+4}]^{1/(4+m)}$, thus

$$|\frac{\eta}{r}| = |n-1|[(4C_r)^m a^{-2m+8} C_\phi^{-4}(\frac{\phi}{H})^{-mn+4n+4m-12}]^{1/(4+m)}. \tag{58}$$

Furthermore, the condition $|\eta/r| < 1$ can explain why during inflation the potential always takes the dominant role, if we rewrite Eq. (58) as $\rho_r/V = 12^{-1}(n-1)^{-1}(\eta/r)(\phi/M)^n$. As $\phi \ll M$, the condition $|\eta/r| < 1$ results in $\rho_r/V < 1$.

The slow-roll parameter c will give a strong constraint on the model, which is

$$|c| = |m + \frac{(1-m)(4+m)}{m-3+2n}| < 4. \tag{59}$$

In the $m = -1, 0, 1, 3$ cases, the exact value of n satisfying Eq. (59) can be seen in Table III. If $m = 1$, this constraint is satisfied by all values of n. But in the $m = -1, 0, 3$ cases, not every n is suitable, e.g if $m = -1, 0$, the standard case $n = 2$ is ruled out; if $m = 3$, the $n = 0$ case is excluded as well.

TABLE III: The suitable range of n for c, and the value of $n_s - 1$, and the suitable n for the red spectrum and the blue spectrum in hybrid-like potential.

m	$\vert c\vert < 4$	$n_s - 1$	red spectrum	blue spectrum
-1	$n > 13/5$, $n < 1$	$\frac{-3\eta}{r}[\frac{3-n}{n-1} + \frac{n}{2(n-1)}\frac{\phi^n}{M^n}]$	$1 < n < 3$	$n < 1, n > 3$
0	$n > 2$, $n < 1$	$\frac{-3\eta}{r}[\frac{-2n+5}{4(n-1)} + \frac{3}{8}\frac{n}{n-1}\frac{\phi^n}{M^n}]$	$1 < n < 5/2$	$n < 1, n > 5/2$
1	any n	$\frac{-3\eta}{r}(\frac{-1}{5} + \frac{3}{10}\frac{n}{n-1}\frac{\phi^n}{M^n})$	none	all n
3	$n > 1$, $n < -7$	$\frac{-3\eta}{r}[\frac{n-7}{7(n-1)} + \frac{3}{14}\frac{n}{n-1}\frac{\phi^n}{M^n}]$	$n < 1, n > 7$	$1 < n < 7$

B. The power spectrum in hybrid-like potential

We now turn to the density perturbations which are caused by the thermal fluctuations. To calculate the magnitude of the thermal fluctuations, we present the definite expressions of T/H and $P_R^{1/2}$ below:

$$\frac{T}{H} = [(4C_\phi C_r)^{-1} a^4 (\frac{\phi}{H})^{m-3+2n}]^{1/(4+m)}, \tag{60}$$

$$P_R^{1/2} = 3^{-1/4}[(4C_r)^{-5m-2} C_\phi^{18} a^{4(3m-6)} (\frac{\phi}{H})^{6(mn-2n-4m+5)}]^{1/(16+4m)}. \tag{61}$$

In Eqs. (60) and (61), a small C_ϕ, which is favored by the observations, will raise the value of T/H, and suppress the amplitude of the power spectrum simultaneously. Moreover, in Eqs. (60) and (61) the explicit dependence on the value of a and ϕ/H should also be noted.

TABLE IV: The value of $dn_s/d\,lnk$, the proper range of n for the negative running, positive running of the spectrum in the $m = -1, 0, 1, 3$ cases in hybrid-like potential.

m	$dn_s/d\,lnk$	negative running	positive running
-1	$\frac{2\eta^2}{(n-1)^2r^2}[\frac{5n-16}{3}(-3n+9+\frac{3n}{2}\frac{\phi^n}{M^n})+\frac{3n^2}{2}\frac{\phi^n}{M^n}]$	$n<3, n>16/5$	$3<n<16/5$
0	$\frac{3\eta^2}{2(n-1)^2r^2}[(n-3)(-2n+5+\frac{3n}{2}\frac{\phi^n}{M^n})+\frac{3n^2}{2}\frac{\phi^n}{M^n}]$	$n<5/2, n>3$	$5/2<n<3$
1	$\frac{6\eta^2}{5(n-1)^2r^2}[\frac{3n-8}{5}(-n+1+\frac{3n}{2}\frac{\phi^n}{M^n})+\frac{3n^2}{2}\frac{\phi^n}{M^n}]$	$n<1, n>8/3$	$2<n<8/3$
3	$\frac{6\eta^2}{7(n-1)^2r^2}[\frac{n}{7}(n-7+\frac{3n}{2}\frac{\phi^n}{M^n})+\frac{3n^2}{2}\frac{\phi^n}{M^n}]$	$0<n<7$	$n>7$

Apply the hybrid-like potential into Eq. (17), then we can give the index of the spectrum

$$n_s - 1 = \frac{-3\eta}{r}(\frac{mn-2n-4m+5}{(4+m)(n-1)} + \frac{3}{2(4+m)}\frac{n}{(n-1)}\frac{\phi^n}{M^n}). \tag{62}$$

Combined Eq. (58) with Eq. (62), the running of the spectrum is

$$\begin{aligned}\frac{dn_s}{d\ln k} &= \frac{6\eta^2}{r^2}[\frac{-mn+4n+4m-12}{(4+m)^2(n-1)^2}(-2n+mn-4m+5+\frac{3n}{2}\frac{\phi^n}{M^n}) \\ &\quad + \frac{3n^2}{2(4+m)(n-1)^2}\frac{\phi^n}{M^n}].\end{aligned} \tag{63}$$

That $n-1=0$ is not a real singularity for the two equations above because from Eq. (58) we can see the η/r expression contains the $n-1$ term too. As $\phi \ll M$, the terms which contain $n\phi^n/M^n$ can be ignored in Eqs. (62) and (63). In Table III, we present the analytic form of $n_s - 1$ in the $m = -1, 0, 1, 3$ cases, the proper n for the red spectrum and blue spectrum as well. In Table IV, we list the proper n for the negative running and also for the positive running of the spectrum. In particular when $m = 1$, the power spectra are always blue. In the following discussion, We briefly consider the cases $n = 0, 2$ specially. When $n = 0$, in the $m = -1, 0, 1$ cases, the spectra are blue with negative running; in the $m = 3$ case, the spectrum is red without running of the spectrum. When $n = 2$, the spectra are red with negative running in the $m = -1, 0$ cases, blue with negative running in the $m = 1$ case, blue with positive running in the $m = 3$ case. In ordinary inflation, when $n = 0$, the spectrum is red and the running is positive; when $n = 2$, the index is blue and no running [22]. In the warm inflation the index of the spectrum and the running can be closer to the observations in some cases [17].

In the hybrid-like potential which has two parameters after n fixed, the discussions on the amplitude of the density perturbations will be more involved than those in the monomial potential. In the $m = 3$ case, the index of a term in Eq. (61) is positive while in the $m = -1, 0, 1$ cases the index of a term is negative. For discussional convenience, we divide the dissipative coefficients into two classes, the $m = 3$ case and the $m = -1, 0, 1$ cases. Further considering the magnitude of a, in the $m = -1, 0, 1$ cases we divide the hybrid-like potentials into three situations $n = 0$, $0 < n < 4$ and $n \geq 4$, while in the $0 < n < 4$ cases we only focus on the $n = 2$ case because of the undetermined a.

When $m = 3$, we notice that the $n = 0$ case is improper for the request from the slow-roll parameter c shown as in Table III. Furthermore, the requirement $T/H > 1$ means:

$$a > (4C_\phi C_r)^{1/4}(\frac{\phi}{H})^{-n/2}, \tag{64}$$

and the condition $|\eta/r| < 1$ yields

$$a < (|n-1|)^{-7/2}(4C_r)^{-3/2}C_\phi^2(\frac{\phi}{H})^{-n/2}. \tag{65}$$

Considering the two equations above, we obtain

$$C_\phi > 4(n-1)^2 C_r. \tag{66}$$

That means though the observations favored small C_ϕ, this model still requires C_ϕ has a lower bound which is at order $\mathcal{O}(C_r)$. Applying Eq. (64) to Eq. (61), the amplitude of the power spectrum gives a constraint as below

$$\frac{\phi}{H} > 1.3 C_r^{1/3} C_\phi^{-1/2} P_R^{-1/3} \approx 10^3 C_r^{1/3} C_\phi^{-1/2}. \tag{67}$$

Through as C_ϕ increases, the lower limit of ϕ/H will decrease. If C_ϕ is of order $\mathcal{O}(C_r)$, ϕ/H should be larger than 10^3. The large ϕ/H means besides $\phi \ll M$, ϕ has a lower bound. Put Eq. (65) into Eq. (55), and we can get

$$\frac{\phi}{T} > 4(n-1)^2 C_r C_\phi^{-1} \frac{\phi}{H}. \tag{68}$$

Assuming that the order of C_ϕ is the same order as C_r, and the value of ϕ/H is larger than 10^3, ϕ/T could be larger than 1, then the model will require a low temperature regime which

could be fulfilled in the $m = 3$ case. However, considering Eqs. (66) and (67), we conclude that C_ϕ and ϕ/H should be matched to satisfy the requirements warm inflation for $m = 3$ as Ref. [14] mentioned.

In the following discussions, we will focus on the $m = -1, 0, 1$ cases, which are similar on the behaviors. Based on the value of a, we divide the potential forms into three cases ($n = 0$, $n \geq 4$, $0 < n < 4$), and in the $0 < n < 4$ regime we will focus on the $n = 2$ case particularly.

When $n = 0$, a is larger than 1 as mentioned above. And in the $m = -1, 0, 1$ cases, $|\eta/r| < 1$ leads to

$$\frac{\phi}{H} > [C_\phi (4C_r)^{-m/4} a^{(m-4)/2}]^{1/(m-3)}, \tag{69}$$

and $T/H > 1$ leads to

$$\frac{\phi}{H} < (4C_\phi C_r a^{-4})^{1/(m-3)}. \tag{70}$$

The two equations above give out the regions that ϕ/H must satisfy. Considering Eqs. (61) and (69), we obtain

$$P_R^{1/2} > 3^{-1/4} (4C_r)^{(-5m+9)/4(m-3)} [a^{3(m-1)} C_\phi^{-3/2}]^{1/(m-3)}. \tag{71}$$

The index of C_ϕ in the above equation is nonnegative when $m = -1, 0, 1$, so is a. As a is larger than or equals to 1, the terms in the square brackets will be larger than or equal to 1 even through C_ϕ is of order $\mathcal{O}(1)$. The other terms on the right hand of Eq. (71) could not make both $P_R^{1/2} \approx 5 \times 10^{-5}$ and Eq. (71) satisfied simultaneously. So when $n = 0$, the warm inflation is not workable.

Now we turn to the $n \geq 4$ cases, where a is smaller than 1. The requirement $T/H > 1$ gives

$$\frac{\phi}{H} > (4C_\phi C_r a^{-4})^{1/(m-3+2n)}, \tag{72}$$

and the above equation shows that ϕ/H is larger than 1. The condition $|\eta/r| < 1$ indicates

$$\frac{\phi}{H} < (|n-1|)^{-4-m} (4C_r)^{-m} C_\phi^4 a^{-8+2m}]^{1/(-mn+4n+4m-12)}. \tag{73}$$

The above two equations can give out a constraint to C_ϕ,

$$C_\phi > [(|n-1|)^{(m-3+2n)/n} (4C_r)^{(m^2+mn+m+4n-12)/(4n+mn)} a^{-2(m-3)/n}. \tag{74}$$

When $n \geq 4$ we give out the constraints from the amplitude of the power spectrum by considering Eqs. (61) and (73) in the $m = -1, 0, 1$ cases respectively,

$$a < 3^{(5n-16)/12}(|n-1|)^{(9-3n)/2}(4C_r)^{(n-2)/12}C_\phi^{(4-n)/2}P_R^{(5n-16)/6}, m = -1; \tag{75}$$

$$a < 3^{(-3+n)/3}(|n-1|)^{(15-6n)/6}(4C_r)^{(n-3)/6}C_\phi^{(4-n)/2}P_R^{(2n-6)/3}, m = 0; \tag{76}$$

$$a < 3^{(8-3n)/12}(|n-1|)^{(1-n)/2}(4C_r)^{(3n-10)/12}C_\phi^{(4-n)/2}P_R^{(3n-8)/6}, m = 1. \tag{77}$$

As n increases, the upper limit of a will decrease, in which the term containing P_R with a positive index is the main contribution. Take $n = 4$ for example where $a^2 = nV_0/M^4$, then for $m = -1$ we have $a < 10^{-6}C_r^{1/6}$, for $m = 0$ we have $a < 6 \times 10^{-7}C_r^{1/6}$, and for $m = 1$ we have $a < 3.4 \times 10^{-7}C_r^{1/6}$. These requirements mean that in order to satisfy the low amplitude of the power spectrum, a should be very small, then V_0 should be very small compared to M^4. As a is small, Eq. (74) shows C_ϕ is larger than a small value. So the constraint to C_ϕ is loose.

To evaluate the suitable working temperature, put Eq. (73) into Eq. (55), and we can get

$$\frac{\phi}{T} > (n-1)^{2n-7}(4C_r)^{(-3m+mn+4n-12)/(4+m)}C_\phi^{(16-4n-mn+4m)/(4+m)}a^{-2}]^{1/(-mn+4n+4m-12)}. \tag{78}$$

When $n \geq 4$, in the $m = -1, 0, 1$ cases, assuming C_ϕ is at order $\mathcal{O}(1)$, that $a < 1$ will lead to $\phi/T > 1$ which means the low temperature regime is needed. Then the $m = -1, 0$ cases can work, while the $m = 1$ case can not.

When $0 < n < 4$, as we do not know the magnitude of a, we only discuss the $n = 2$ case specifically. When $m = -1, 0$, the requirements from the slow-roll parameter c exclude the $n = 2$ case just as shown in Table III, then we only need to discuss the $m = 1$ case. When $m = 1$, the requirement $T/H > 1$ leads to

$$a > (4C_\phi C_r)^{1/4}(\frac{\phi}{H})^{-1/2}, \tag{79}$$

and the condition $|\eta/r| < 1$ gives out

$$a < (4C_r)^{-1/6}C_\phi^{2/3}(\frac{\phi}{H})^{1/3}. \tag{80}$$

These two equations above can give out $\phi/H > (4C_r C_\phi^{-1})^{1/2}$. The amplitude of the spectrum also gives a constraint

$$\frac{\phi}{H} > (12^{-1} C_r^{-1} C_\phi)^{1/2} P_R^{-1} \approx 10^8 (C_r^{-1} C_\phi)^{1/2}, \tag{81}$$

which shows that we need large ϕ/H for a low amplitude of the power spectrum. From Eqs. (55) and (61),

$$\frac{\phi}{T} > 4^{1/3}(n-1)^{-2/3}(\frac{\phi}{H})^{-1/3}, \tag{82}$$

where we can know that the large value of ϕ/H will make ϕ/T smaller than 1, and the regime of the low temperature excludes the $m = 1$ case. Noticing that the requirement from the slow-roll parameter c excludes the $m = -1, 0$ cases, the scenario does not work well when $n = 2$.

In hybrid-like model, when $m = 3$, it requires the suitable matching between C_ϕ and ϕ/H to work. When $n \geq 4$, it will need very small a, large ϕ/H and large n to work in the $m = -1, 0$ cases. When $m = -1, 0, 1$, warm inflation can not satisfy the observations of low amplitude of power spectrum in the $n = 0$ case. When $n \geq 4$ it can not available in the $m = 1$ case for the workable temperature regime. And it cannot work in the $n = 2$ case for the sake of parameter c or the low working temperature regime when $m = -1, 0, 1$.

VI. CONCLUSION

Although, the slow-roll ordinary inflation is the dominant paradigm in the very early universe. This mechanism can be replaced by the warm inflation in which we can not neglect the effect from the radiation due to the interactions between the inflaton and other fields. The warm inflation scenario can alleviate the η problem, and its thermal fluctuations are responsible for the generation of the density perturbations if whose amplitude exceeds than that of quantum fluctuations.

In this work, based on the exact calculational results of the dissipative coefficient in Refs. [8, 12, 13], we extended the various cases of the dissipative coefficient to a general form $\Gamma = C_\phi T^m/\phi^{m-1}$ as a toy model. And we focused on the strong dissipative regime where $r \gg 1$. To examine the conditions which constrain the realization of warm inflation, firstly

we gave out the slow-roll conditions and the power spectrum in the toy model which agree with the previous results, then we took two specific potential forms (the monomial potential and the hybrid-like potential) to discuss the slow-roll conditions and the power spectrum in detail. The slow-roll conditions are different from the ordinary inflation, which mean not only the number of the slow-roll parameters are increased but also the allowed regimes of the slow-roll parameters are enlarged which may solve can alleviatethe η problem. The amplitude of the power spectrum is not only depend on the Hubble parameter and the potential, but also the temperature and the dissipative coefficient. However, we considered $\phi \leq m_{pl}$ to constrain the value of the inflaton field and $C_\phi < 10^4$ to constrain the number of the inflaton fields. based on the rationality in both theory and observation. The field regime $\phi \leq m_{pl}$ is interested as in the ordinary inflation the monomial potential has the causal problem. The regime $C_\phi < 10^4$ is concerned because some warm inflation models need too large number of fields to realize. Under these strong assumptions, in the monomial potential, the $m = 0$ case is a workable model; and when $2 \leq n < 10/3$, the $m = 1$ can work too. In the hybrid-like potential, the $m = 3$ case is workable, and when $n \geq 4$ the $m = -1, 0$ cases can work as well. It seems that the cases that $m = 0, 3$ are worth further research because they are possibly permissible in warm inflation framework. And subsequent work on the details of warm inflation are worthy to be investigated too. After all, it is a workable scenario which is different from ordinary inflation.

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